Robotics - Single view, Epipolar geometry, Image Features

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Outline

Outline

Pin hole model - Recall

The intrinsic camera matrix

or *calibration matrix*

$$\mathbf{K} = \begin{bmatrix} f_x & s & \mathbf{c}_x \\ 0 & f_y & \mathbf{c}_y \\ 0 & 0 & 1 \end{bmatrix}$$

- f_x, f_y: focal lenght (in pixels)

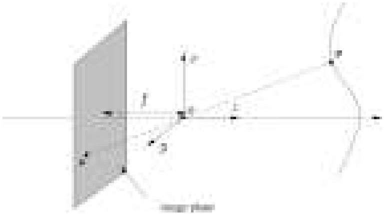
  $f_x/f_y = s_x/s_y = a$: aspect ratio
- s: skew factor

  pixel not orthogonal

  usually 0 in modern cameras
- $\mathbf{c}_x$, $\mathbf{c}_y$: principal point (in pixel)

  usually $\neq$ half image size due to misalignment of CCD

Projection



$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} \mathbf{K} & \mathbf{0} \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ W \end{bmatrix}$$

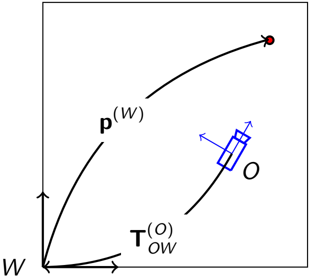
$$\begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} \frac{f_x X}{Z} + \mathbf{c}_x \\ \frac{f_y Y}{Z} + \mathbf{c}_y \end{bmatrix}$$

Points in the world

CONSIDER

- $$\mathbf{p}^{(\mathbf{I}')} = \begin{bmatrix} f_x & s & \mathbf{c}_x & 0 \\ 0 & f_y & \mathbf{c}_y & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \mathbf{P}^{(O)}$$
- $\mathbf{P}^{(O)} = \mathbf{T}_{OW}^{(O)} \mathbf{P}^{(W)}$

  extrinsic camera matrix



Points in the world

Consider

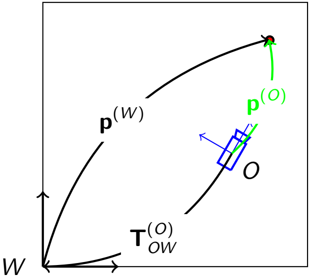
- $\mathbf{p}^{(\mathbf{I}')} = \begin{bmatrix} f_x & s & \mathbf{c}_x & 0 \\ 0 & f_y & \mathbf{c}_y & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \mathbf{P}^{(O)}$
- $\mathbf{P}^{(O)} = \mathbf{T}_{OW}^{(O)} \mathbf{P}^{(W)}$

  extrinsic camera matrix

One step

- $\mathbf{p}^{(\mathbf{I}')} = \begin{bmatrix} \mathbf{K} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{R} & \mathbf{t} \\ \mathbf{0} & 1 \end{bmatrix} \mathbf{P}^{(W)}$
- $\pi = \begin{bmatrix} \mathbf{KR} & \mathbf{Kt} \end{bmatrix}$

  complete projection matrix

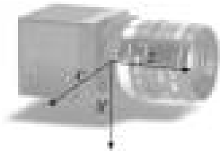


Note

- $\mathbf{R}$ is $\mathbf{R}_{OW}^{(O)}$
- $\mathbf{t}$ is $\mathbf{t}_{OW}^{(O)}$
- i.e., the position and orientation of W in O

Note on camera reference system

Camera reference system



- z: front
- y: down

World reference system

- x: front
- z: up

Rotation of O w.r.t. W

- Rotate around y of 90°

  z' front

- Rotate around z' of -90°

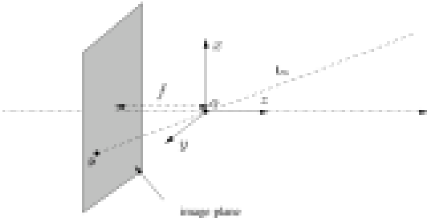
  y'' point down

- $\mathbf{R}_{WO}^{(W)} = \mathbf{R}_y(90^\circ)\,\mathbf{R}_z(-90^\circ)$

- $$\mathbf{R}_{WO}^{(W)} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix}$$

- $$\mathbf{R}_{OW}^{(O)} = \mathbf{R}_{WO}^{(W)\,T} = \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ 1 & 0 & 0 \end{bmatrix}$$

Interpretation line



Given

- $\mathbf{p}^{(\mathbf{I})} = \left[u, v\right]^T$: point in image (pixel)

Calculate $\mathbf{P}^{(O)}$?

- No, only $\mathbf{l}_{Po}$: interpretation line
- $\forall \mathbf{P}_i^{(O)} \in \mathbf{l}_{Po}$ image is $\mathbf{p}^{(\mathbf{I})}$

Calculate $\mathbf{l}_{Po}$

- 3D lines not coded in 3D

  remember duality points $\leftrightarrow$ planes
- $\mathbf{p}^{(\mathbf{I}')} = \left[\mathbf{K} \quad \mathbf{0}\right] \mathbf{P}^{(O)}$
- $\mathbf{p}^{(\mathbf{I}')} = \left[\mathbf{K} \quad \mathbf{0}\right] \left[X,\ Y,\ Z,\ W\right]^T$
- $\mathbf{p}^{(\mathbf{I}')} = \mathbf{K} \left[X,\ Y,\ Z\right]^T$

  W "cancelled" by zeros fourth column
- $\mathbf{d}^{(\mathbf{O})} = \mathbf{K}^{-1} \left[u, v, 1\right]^T$
- $\overline{\mathbf{d}}^{(\mathbf{O})} = \mathbf{d}^{(\mathbf{O})} / \|\mathbf{d}^{(\mathbf{O})}\|$: unit vector
- $\mathbf{P}_i^{(\mathbf{O})} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} + \lambda \begin{bmatrix} \overline{\mathbf{d}}^{(\mathbf{O})} \\ 0 \end{bmatrix}, \lambda > 0$

  $\mathbf{l}_{Po}$ in parametric form

Interpretation line & Normalized image plane

Interpretation line direction

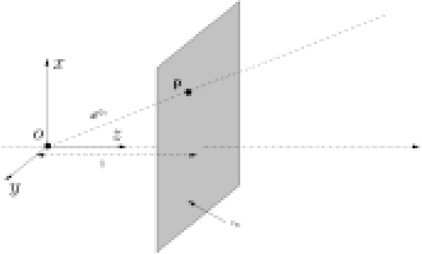
- $\mathbf{d}^{(O)} = \mathbf{K}^{-1}\left[u, v, 1\right]^T$
- $\mathbf{K}^{-1} = \begin{bmatrix} 1/f_x & 0 & -\mathbf{c}_x/fx & 0 \\ 0 & 1/f_y & -\mathbf{c}_y/fy & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$

  assume skew $= 0$
- $\mathbf{d}^{(O)} = \left[\frac{u-\mathbf{c}_x}{fx}, \frac{v-\mathbf{c}_y}{fy}, 1\right]^T$
- $\mathbf{P}^{(O)}_{\lambda=\|\mathbf{d}^{(O)}\|} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} + \begin{bmatrix} \mathbf{d}^{(O)} \\ 0 \end{bmatrix}$

  lies on π_N
- If $u = \mathbf{c}_x$, $v = \mathbf{c}_y$, $\mathbf{d}$ is

  the *principal direction*

Normalized image plane



- Distance 1 from the optical center
- Independent of camera intrinsic

Given a cartesian point $\mathbf{P}^{(O)} = \left[X, Y, Z\right]^T$

$$\mathbf{P}^{(O)}_{\pi_N} = \left[X/Z, Y/Z, 1\right]^T$$

Interpretation line in the world - 1

Consider

- Interpretation line in camera coordinate

$$\mathbf{P}_i^{(\mathbf{O})} = \begin{bmatrix} \mathbf{0} \\ 1 \end{bmatrix} + \lambda \begin{bmatrix} \overline{\mathbf{d}}^{(\mathbf{O})} \\ 0 \end{bmatrix}$$

- Interpretation line in world coordinate

$$\begin{aligned} \mathbf{P}_i^{(\mathbf{W})} &= \begin{bmatrix} \mathbf{R}_{OW}^{(O)\,T} & -\mathbf{R}_{OW}^{(O)\,T}\mathbf{t}_{OW}^{(O)} \\ \mathbf{0} & 1 \end{bmatrix} \left(\begin{bmatrix} \mathbf{0} \\ 1 \end{bmatrix} + \lambda \begin{bmatrix} \overline{\mathbf{d}}^{(\mathbf{O})} \\ 0 \end{bmatrix} \right) \\ &= \begin{bmatrix} -\mathbf{R}_{OW}^{(O)\,T}\mathbf{t}_{OW}^{(O)} \\ 1 \end{bmatrix} + \begin{bmatrix} \lambda\,\mathbf{R}_{OW}^{(O)\,T}\,\overline{\mathbf{d}}^{(\mathbf{O})} \\ 0 \end{bmatrix} \\ &= \mathbf{O}^{(W)} + \lambda\,\overline{\mathbf{d}}^{(\mathbf{W})} \end{aligned}$$

- Camera center in world coordinate + direction rotated as world reference

Interpretation line in the world - 2

Consider

- Interpretation line in world coordinate

$$\begin{aligned} \mathbf{P}_i^{(\mathbf{W})} &= \lambda \, \mathbf{R}_{OW}^{(O)\,T} \, \overline{\mathbf{d}}^{(\mathbf{O})} - \mathbf{R}_{OW}^{(O)\,T} \mathbf{t}_{OW}^{(O)} \\ &= \lambda \, \mathbf{R}_{OW}^{(O)\,T} \, \mathbf{K}^{-1} \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} - \mathbf{R}_{OW}^{(O)\,T} \mathbf{t}_{OW}^{(O)} \end{aligned}$$

- Complete projection matrix

$$\boldsymbol{\pi} = \begin{bmatrix} \mathbf{K}\mathbf{R}_{OW}^{(O)} & \mathbf{K}\mathbf{t}_{OW}^{(O)} \end{bmatrix} = \begin{bmatrix} \mathbf{M} & \mathbf{m} \end{bmatrix}$$

- $\mathbf{M}^{-1} = \mathbf{R}_{OW}^{(O)\,T} \mathbf{K}^{-1}$: direction in world coordinate given pixel coordinate
- $-\mathbf{M}^{-1}\mathbf{m} = -\mathbf{R}_{OW}^{(O)\,T} \, \mathbf{K}^{-1} \, \mathbf{K} \, \mathbf{t}_{OW}^{(O)} = -\mathbf{R}_{OW}^{(O)\,T} \, \mathbf{t}_{OW}^{(O)} = \mathbf{t}_{WO}^{(W)}$:

  camera center in world coordinate $\mathbf{O}^{(W)}$

Principal ray

Interpretation line of principal point

- $\mathbf{d}^{(\mathbf{O})} = \mathbf{K}^{-1} \begin{bmatrix} \mathbf{c}_x \\ \mathbf{c}_y \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

  z-axis of the camera reference system

- $\mathbf{d}^{(\mathbf{W})} = \mathbf{R}_{OW}^{(O)\ T} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \mathbf{M}^{-1} \begin{bmatrix} \mathbf{c}_x \\ \mathbf{c}_y \\ 1 \end{bmatrix}$

  z-axis of the camera in world reference system

- $\mathbf{P}_i^{(\mathbf{W})} = \lambda\, \mathbf{M}^{-1} \begin{bmatrix} \mathbf{c}_x \\ \mathbf{c}_y \\ 1 \end{bmatrix} - \mathbf{M}^{-1}\mathbf{m}$

  parametric line of z-axis of the camera in world reference system

Vanishing points & Origin

Vanishing points

- $\mathbf{V}_x^{(\mathbf{W})} = \begin{bmatrix}1, 0, 0, 0\end{bmatrix}^T$
- $\mathbf{V}_y^{(\mathbf{W})} = \begin{bmatrix}0, 1, 0, 0\end{bmatrix}^T$
- $\mathbf{V}_z^{(\mathbf{W})} = \begin{bmatrix}0, 0, 1, 0\end{bmatrix}^T$

Projection on the image

- $\mathbf{p}^{(\mathbf{I}')} = \begin{bmatrix}\mathbf{M} & \mathbf{m}\end{bmatrix} \begin{bmatrix}\mathbf{d} \\ 0\end{bmatrix} = \mathbf{Md}$
- $\mathbf{p}_x^{(\mathbf{I}')} = \begin{bmatrix}\mathbf{M} & \mathbf{m}\end{bmatrix} \mathbf{V}_x^{(\mathbf{W})} = \mathbf{M}^{(1)}$
- $\mathbf{p}_x^{(\mathbf{I}')} = \begin{bmatrix}\mathbf{M} & \mathbf{m}\end{bmatrix} \mathbf{V}_y^{(\mathbf{W})} = \mathbf{M}^{(2)}$
- $\mathbf{p}_z^{(\mathbf{I}')} = \begin{bmatrix}\mathbf{M} & \mathbf{m}\end{bmatrix} \mathbf{V}_z^{(\mathbf{W})} = \mathbf{M}^{(3)}$

Origin

- $\mathbf{O}^{(\mathbf{W})} = \begin{bmatrix}0, 0, 0, 1\end{bmatrix}^T$

Projection on the image

- $\mathbf{p}^{(\mathbf{I}')} = \begin{bmatrix}\mathbf{M} & \mathbf{m}\end{bmatrix} \begin{bmatrix}\mathbf{0} \\ 1\end{bmatrix} = \mathbf{m}$

Vanishing points & Origin

Vanishing points

- $\mathbf{V}_x^{(\mathbf{W})} = \begin{bmatrix}1, 0, 0, 0\end{bmatrix}^T$
- $\mathbf{V}_y^{(\mathbf{W})} = \begin{bmatrix}0, 1, 0, 0\end{bmatrix}^T$
- $\mathbf{V}_z^{(\mathbf{W})} = \begin{bmatrix}0, 0, 1, 0\end{bmatrix}^T$

Projection on the image

- $\mathbf{p}^{(\mathbf{I}')} = \begin{bmatrix}\mathbf{M} & \mathbf{m}\end{bmatrix} \begin{bmatrix}\mathbf{d} \\ 0\end{bmatrix} = \mathbf{Md}$
- $\mathbf{p}_x^{(\mathbf{I}')} = \begin{bmatrix}\mathbf{M} & \mathbf{m}\end{bmatrix} \mathbf{V}_x^{(\mathbf{W})} = \mathbf{M}^{(1)}$
- $\mathbf{p}_x^{(\mathbf{I}')} = \begin{bmatrix}\mathbf{M} & \mathbf{m}\end{bmatrix} \mathbf{V}_y^{(\mathbf{W})} = \mathbf{M}^{(2)}$
- $\mathbf{p}_z^{(\mathbf{I}')} = \begin{bmatrix}\mathbf{M} & \mathbf{m}\end{bmatrix} \mathbf{V}_z^{(\mathbf{W})} = \mathbf{M}^{(3)}$

Origin

- $\mathbf{O}^{(\mathbf{W})} = \begin{bmatrix}0, 0, 0, 1\end{bmatrix}^T$

Projection on the image

- $\mathbf{p}^{(\mathbf{I}')} = \begin{bmatrix}\mathbf{M} & \mathbf{m}\end{bmatrix} \begin{bmatrix}\mathbf{0} \\ 1\end{bmatrix} = \mathbf{m}$

Note

- Col 1 of $\boldsymbol{\pi}$ is image of x vanishing point
- Col 2 of $\boldsymbol{\pi}$ is image of y vanishing point
- Col 3 of $\boldsymbol{\pi}$ is image of w vanishing point
- Col 4 of $\boldsymbol{\pi}$ is image of $\mathbf{O}^{(\mathbf{W})}$

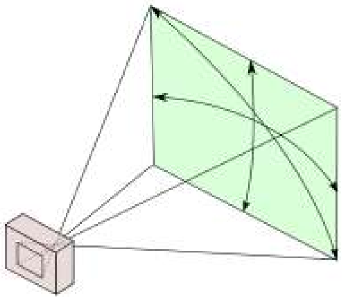
Angle of View

Given

- Image size: $[w, h]$
- Focal lenght: f_x (assume $f_x = f_y$)

Angle of view

- $\theta = 2\text{atan2}(w/2, f_x)$
- $\theta < 180^\circ$



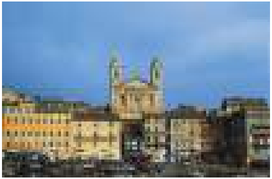
Examples

14mm

20mm

28mm

35mm



50mm

From past exam

Ex. 4 - 20 November 2006

Problem

- Given $\boldsymbol{\pi} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 3 & 1 & 0 & 4 \\ 1 & 2 & 3 & 1 \end{bmatrix}$
- Where is the camera center in world reference frame?

From past exam

Ex. 4 - 20 November 2006

Problem

- Given $\boldsymbol{\pi} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 3 & 1 & 0 & 4 \\ 1 & 2 & 3 & 1 \end{bmatrix}$
- Where is the camera center in world reference frame?

Solution

- $\boldsymbol{\pi} = \begin{bmatrix} \mathbf{M} & \mathbf{m} \end{bmatrix}$
- $\mathbf{O}^{(W)} = -\mathbf{M}^{-1}\mathbf{m}$
- $\mathbf{M}^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ 5/3 & -2/3 & 1/3 \end{bmatrix}$
- $\mathbf{O}^{(W)} = \begin{bmatrix} -1 \\ -1 \\ 2/3 \end{bmatrix}$

Outline

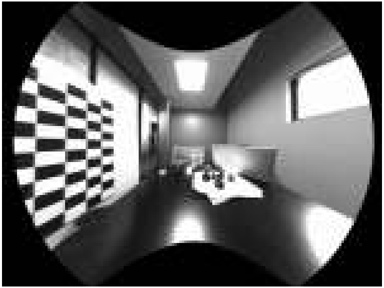
Distortion

Distortion

- Deviation from rectilinear projection
- Lines in scene don't remains lines in image

Original image

Corrected



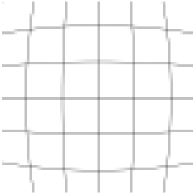
Distortion

Distortion

- Can be irregular
- Most common is *radial* (radially symmetric)

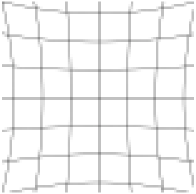
Radial Distortion

Barrel distortion



Magnification decrease with distance from optical axis

Pincushion distortion



Magnification increase with distance from optical axis

Brown distortion model

CONSIDER

- $\mathbf{P}^{(O)} = \begin{bmatrix} X, & Y, & Z \end{bmatrix}^T$ in camera reference system
- Calculate $\mathbf{p}^{(\mathbf{I})} = \begin{bmatrix} x, & y, & 1 \end{bmatrix}^T = \begin{bmatrix} X/Z, & Y/Z, & 1 \end{bmatrix}^T$ on the normalized image plane

DISTORTION MODEL

- $\tilde{\mathbf{p}}^{(\mathbf{I})} = \left(1 + \mathbf{k}_1 r^2 + \mathbf{k}_2 r^4 + \mathbf{k}_3 r^6\right) \mathbf{p}^{(\mathbf{I})} + d_x$
- $r^2 = x^2 + y^2$: distance wrt optical axis $(0, 0)$
- $d_x = \begin{bmatrix} 2p_1xy + p_2(r^2 + 2x^2) \\ p_1(r^2 + 2y^2) + 2p_2xy \end{bmatrix}$: tangential distortion compensation

IMAGE COORDINATE

- $\tilde{\mathbf{p}}^{(\mathbf{I}')} = \mathbf{K}\tilde{\mathbf{p}}^{(\mathbf{I})}$: pixel coordinate of $\mathbf{P}^{(O)}$ considering distortion

Brown distortion model

From image points

- $\tilde{\mathbf{p}}^{(\mathbf{I'})}$ in image (pixel)
- Calculate $\tilde{\mathbf{p}}^{(\mathbf{I})} = \mathbf{K}^{-1}\tilde{\mathbf{p}}^{(\mathbf{I'})} = \left[x,\ y,\ 1\right]^T$ on the (distorted) normalized image plane
- Undistort: $\mathbf{p}^{(\mathbf{I})} = dist^{-1}(\tilde{\mathbf{p}}^{(\mathbf{I})})$
- Image projection: $\mathbf{p}^{(\mathbf{I'})} = \mathbf{K}\mathbf{p}^{(\mathbf{I})}$

Evaluation of $dist^{-1}(\cdot)$

- No analytic solution
- Iterative solution ($N = 20$ is enough):

1: $\mathbf{p}^{(\mathbf{I})} = \tilde{\mathbf{p}}^{(\mathbf{I})}$: initial guess
2: **for** $i = 1$ to N **do**
3: $\quad r^2 = x^2 + y^2,\ k_r = \left(1 + \mathbf{k}_1 r^2 + \mathbf{k}_2 r^4 + \mathbf{k}_3 r^6\right),\ d_x = \begin{bmatrix} 2p_1xy + p_2(r^2 + 2x^2) \\ p_1(r^2 + 2y^2) + 2p_2xy \end{bmatrix}$
4: $\quad \mathbf{p}^{(\mathbf{I})} = \left(\tilde{\mathbf{p}}^{(\mathbf{I})} - d_x\right)/k_r$
5: **end for**

Outline

Camera calibration

Intrinsic calibration

- Find parameters of $\mathbf{K}$
- Nominal values of optics are not suitable
- Differences between different exemplar of same camera/optic system
- Include distortion coefficient estimation

Extrinsic calibration

- Find parameters of $\boldsymbol{\pi} = \begin{bmatrix} \mathbf{M} & \mathbf{m} \end{bmatrix}$
- i.e., find $\mathbf{K}$ and $\mathbf{R}$, $\mathbf{t}$

Camera calibration - Approaches

Calibration

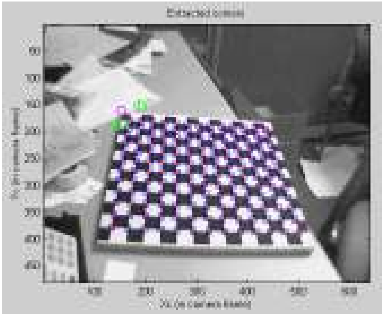
- Very large literature!
- Different approaches

Methods

- Based on correspondances
- Need for a pattern

Known 3D pattern

Planar pattern



Camera calibration - Formulation

Formulation

- $\mathbf{M}_i$: model points on the pattern
- $\mathbf{p}_{ij}$: observation of model point i in image j
- $\mathbf{p} = \left[f_x, f_y, s, \cdots k_1, k_2, \cdots \right]^T$: intrinsic parameters
- $\mathbf{R}_j, \mathbf{t}_j$: pose of the patter wrt camera reference frame j

  i.e., $\mathbf{R}^{(C)}_{CP\,j}, \mathbf{t}^{(C)}_{CP\,j}$
- $\hat{\mathbf{m}}(\mathbf{p}, \mathbf{R}_j, \mathbf{t}_j, \mathbf{M}_i)$: estimated projection of $\mathbf{M}_i$ in image j.

Estimation

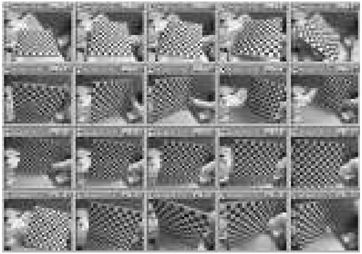
- $\underset{\mathbf{p}, \mathbf{R}_j, \mathbf{t}_j}{\operatorname{argmin}} \sum_j \sum_i \mathbf{p}_{ij} - \mathbf{m}_{ij}(\mathbf{p}, \mathbf{R}_j, \mathbf{t}_j, \mathbf{M}_i)$: observation of model point i in image j
- Gives both *intrinsic* (unique) and *extrinsic* (one for each image) calibration

Z. Zhang, "A flexible new technique for camera calibration", 2000

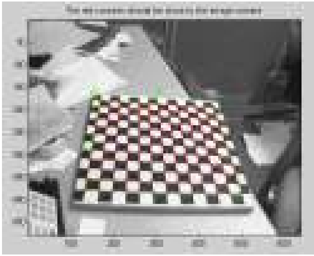
Heikkila, Silvén, "A Four-step Camera Calibration Procedure with Implicit Image Correction", 1997

Camera Calibration Toolbox for Matlab

Collect images

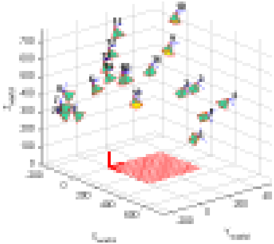


Automatic corners identification



Find chessboard external corners

Calibration



Camera Calibration Toolbox for Matlab - *http://www.vision.caltech.edu/bouguetj/calib_doc/*

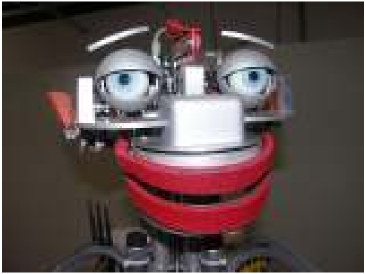
Outline

Epipolar geometry introduction

Epipolar geometry

- Projective geometry between two views
- Independent of scene structure
- Depends only on
  - Cameras parameters
  - Cameras relative position
- Two views
  - Simultaneously (stereo)
  - Sequentially (moving camera)

Bumblebee camera



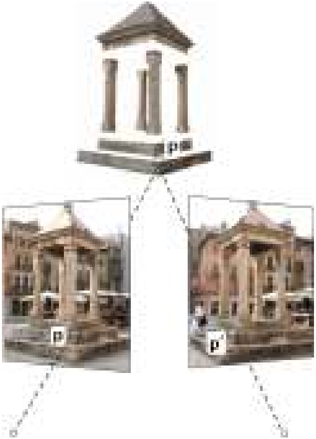
Robot head with two cameras

Correspondence

CONSIDER

- $\mathbf{P}$ a 3D point in the scene
- Two cameras with $\boldsymbol{\pi}$ and $\boldsymbol{\pi}'$
- $\mathbf{p} = \boldsymbol{\pi}\mathbf{P}$ image on first camera
- $\mathbf{p}' = \boldsymbol{\pi}'\mathbf{P}$ image on second camera
- $\mathbf{p}$ and $\mathbf{p}'$: images of the same point

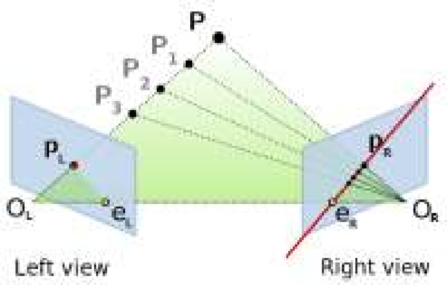
  $\rightarrow$ *correspondence*



CORRESPONDENCE GEOMETRY

- $\mathbf{p}$ on first image
- How $\mathbf{p}'$ is constrained by $\mathbf{p}$?

Correspondences and epipolar geometry



Suppose (1)

- $\mathbf{P}$, a 3D point imaged in two views
- $\mathbf{p}_L$ and $\mathbf{p}_R$ image of $\mathbf{P}$
- $\mathbf{P}$, $\mathbf{p}_L$, $\mathbf{p}_R$, O_L, O_R are coplanar on π
- π is the *epipolar plane*

Suppose (2)

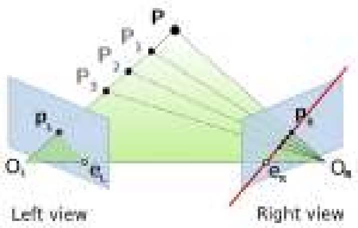
- $\mathbf{P}$ is unknown
- $\mathbf{p}_L$ is known
- Where is $\mathbf{p}_R$?

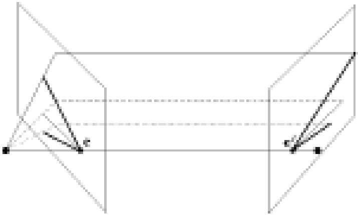
  or how is constrained $\mathbf{p}_R$?
- $\mathbf{P}_i = O_L + \lambda \mathbf{d}_{\mathbf{p}_L O_L}$ is the interpretation line of $\mathbf{p}_L$
- $\mathbf{p}_R$ lies on a line:

  intersection of π with the 2^{nd} image

  $\rightarrow$ *epipolar line*

Epipolar geometry - Definitions





Base line

- Line joining O_L and O_R

Epipoles $(\mathbf{e}_L, \mathbf{e}_R)$

- Intersection of base line with image planes
- Projection of camera centres on images
- Intersection of all epipolar lines

Epipolar line

- Intersection of epipolar plane with image plane

Epipolar plane

- A plane containing the baseline
- It's a pencil of planes
- Given an *epipolar line* is possible to identify a unique epipolar plane

Epipolar constraints

Correspondences problem

- Given $\mathbf{p}_L$ in one image
- Search on second image along the *epipolar line*
- 1D search!
- A point in one image "generates" a line in the second image

Correspondences example

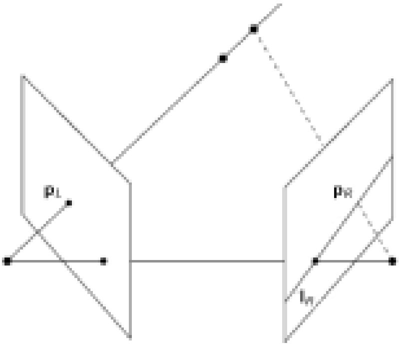
The fundamental matrix F

Epipolar geometry

- Given $\mathbf{p}_L$ on one image
- $\mathbf{p}_R$ lies on $\mathbf{l}_R$, i.e. the *epipolar line*
- $\mathbf{p}_R \in \mathbf{l}_R \leftrightarrow \mathbf{p}_R^T \mathbf{l}_R = 0$
- Thus, there is a map $\mathbf{p}_L \to \mathbf{l}_R$

The fundamental matrix F

- $\mathbf{l}_R = \mathbf{F}\,\mathbf{p}_L$
- $\mathbf{F}$ is the fundamental matrix
- $\mathbf{p}_R \in \mathbf{l}_R \leftrightarrow \mathbf{p}_R^T \mathbf{F}\,\mathbf{p}_L = 0$



The fundamental matrix **F** properties

Properties

- If $\mathbf{p}_L$ correspond to $\mathbf{p}_R$ $\rightarrow \mathbf{p}_L \mathbf{F} \mathbf{p}_R = 0$, necessary condition for correspondence
- If $\mathbf{p}_L \mathbf{F} \mathbf{p}_R = 0$ interpretation lines (a.k.a. *viewing ray*) are coplanar
- **F** is a 3x3 matrix
- $\det(\mathbf{F}) = 0$
- $\text{rank}(\mathbf{F}) = 2$
- **F** has 7 dof (1 homogeneous, 1 rank deficient)
- $\mathbf{l}_R = \mathbf{F}\,\mathbf{p}_L$, $\mathbf{l}_L = \mathbf{F}^T\,\mathbf{p}_R$
- $\mathbf{F}\mathbf{e}_L = 0$, $\mathbf{F}^T\mathbf{e}_R = 0$, i.e.: epipoles are the right null vector of **F** and $\mathbf{F}^T$

  Proof: $\forall \mathbf{p}_L \neq \mathbf{e}_L, \quad \mathbf{l}_R = \mathbf{F}\,\mathbf{p}_L$ and $\mathbf{e}_R \in \mathbf{l}_R \;\rightarrow \forall \mathbf{p}_L \quad \mathbf{e}_R^T \mathbf{F}\,\mathbf{p}_L = 0 \rightarrow \mathbf{F}^T \mathbf{e}_R = 0$

F calculus

FROM CALIBRATED CAMERAS

- $\boldsymbol{\pi}_L$ and $\boldsymbol{\pi}_R$ are known
- $\mathbf{F} = [\mathbf{e}_R]_\times \boldsymbol{\pi}_R \boldsymbol{\pi}_L^+$

  where

  - $\mathbf{e}_R = \boldsymbol{\pi}_R \mathbf{O}_L = \boldsymbol{\pi}_R \left(-\mathbf{M}_L^{-1}\mathbf{m}_L\right)$
  - $\boldsymbol{\pi}_L^+ = \boldsymbol{\pi}_L^T \left(\boldsymbol{\pi}_L \boldsymbol{\pi}_L^T\right)^{-1}$: pseudo-inverse
  - $\mathbf{a} \times \mathbf{b} = [\mathbf{a}]_\times \mathbf{b}$ where $[\mathbf{a}]_\times = \begin{bmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{bmatrix}$

CALIBRATED CAMERAS WITH $\mathbf{K}$ AND $\mathbf{R}, \mathbf{t}$

- $\boldsymbol{\pi}_L = \mathbf{K}_L \begin{bmatrix} \mathbf{I} & \mathbf{0} \end{bmatrix}$: origin in the left camera
- $\boldsymbol{\pi}_R = \mathbf{K}_R \begin{bmatrix} \mathbf{R} & \mathbf{t} \end{bmatrix}$
- $\mathbf{F} = \mathbf{K}_R^{-T} [\mathbf{t}]_\times \mathbf{R}\, \mathbf{K}_L^{-1}$
- Special forms with pure translations, pure rotations, ...

F estimation - procedure sketch

From uncalibrated images

- Get point correspondances ("by hand" or automatically)
- Compute **F** by consider that
  - $\mathbf{p}_R^T \mathbf{F} \mathbf{p}_L = 0$
  - At least 7 correspondances are needed

    but the 8-point algorithm is the simplest
  - Impose rank($\mathbf{F}$) = 2

Details

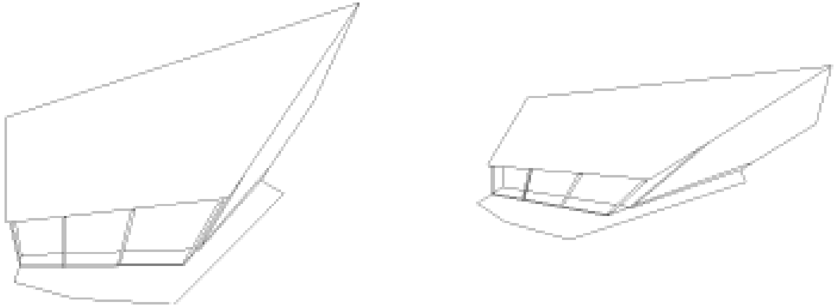
"Multiple View Geometry in computer vision"

Hartley Zisserman. Chapters 9,10,11,12.

Projective reconstruction

F IS NOT UNIQUE

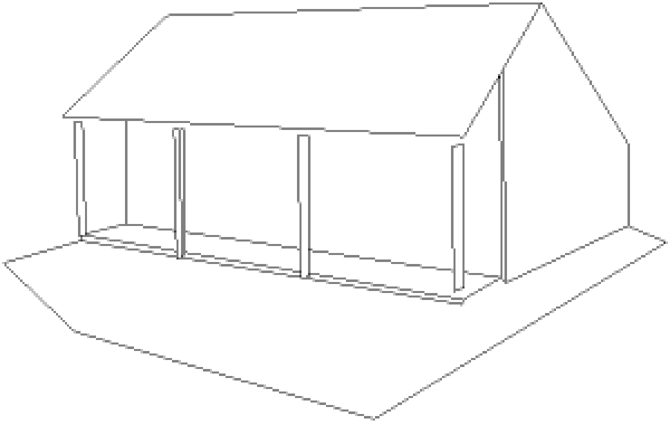
- If estimated by correspondences
- Without any additional constraints allow at least a projective reconstruction



Metric reconstruction and reconstruction

Additional constraints

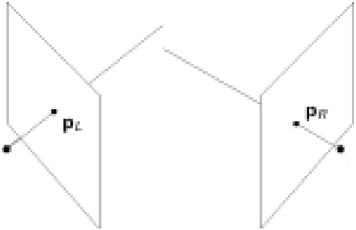
- Parallelism, measures of some points, ...
- $\rightarrow$ allow affine/similar/metric reconstruction



Triangulation

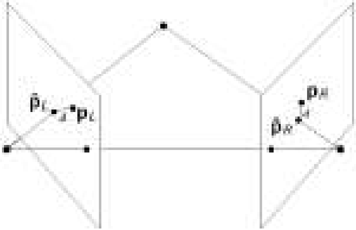
Suppose

- $\mathbf{p}_R$ and $\mathbf{p}_L$ are correspondent points
- π_L and π_R are known
- Due to noise is possible that interpretation lines don't intersect
- $\mathbf{p}_L \mathbf{F} \mathbf{p}_R \neq 0$



3D point computation

- $\underset{\hat{\mathbf{p}}_L, \hat{\mathbf{p}}_R}{\text{argmin}} \quad d(\hat{\mathbf{p}}_L, \mathbf{p}_L)^2 + d(\hat{\mathbf{p}}_R, \mathbf{p}_R)^2$
- subject to $\hat{\mathbf{p}}_L \mathbf{F} \hat{\mathbf{p}}_R = 0$



Outline

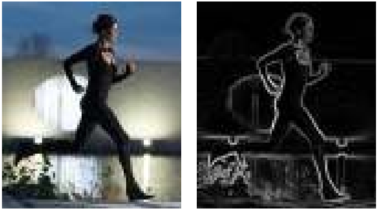
Features in image

What is a feature?

- No common definition
- Depends on problem or application
- Is an *interesting part* of the image

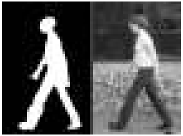
Types of features

- Edges
  - Boundary between regions
- Corners / interest points
  - Edge intersection
  - Corners
  - Point-like features
- Blobs
  - Smooth areas that define regions



Edges

Corners



Blob

Black & White threshold

Thresholding

- On a gray scale image $I(u, v)$
- If $I(u, v) > T \quad I'(u, v) = white$
- else $I'(u, v) = black$

Properties

- Simplest method of image segmentation
- Critical point: threshold T value
  - Mean value of $I(u, v)$
  - Median value of $I(u, v)$
  - (Local) adaptive thresholding

Filtering

KERNEL MATRIX FILTERING

- Given an image $I(i,j), i = 1 \cdots h, j = 1 \cdots w$
- A kernel $H(k,z), k = 1 \cdots r, z = 1 \cdots c$
- $I'(i,j) = \sum_k \sum_z I(i - \lfloor r/2 \rfloor + k - 1, j - \lfloor c/2 \rfloor + z - 1) * H(k,z)$
- special cases on borders

$I(i,j)$

2	2	2	3
2	1	3	3
2	2	1	2
1	3	2	2

$H(k,z)$

1	1	1
-1	2	1
-1	-1	1

2	2	2	3
-2	2	3	3
-2	-2	1	2
1	3	2	2

$I'(2,2)$

5	4	4	-2
9	6		

FINAL RESULT

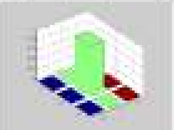
5	4	4	-2
9	6	14	5
11	7	6	5
9	12	8	5

Filter Examples - 1

IDENTITY

*

0	0	0
0	1	0
0	0	0

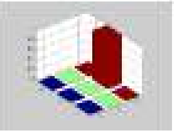


=

TRANSLATION

*

0	0	0
0	0	1
0	0	0



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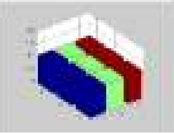
Filter Examples - 2

AVERAGE (3 × 3)

$$* \frac{1}{9} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} =$$

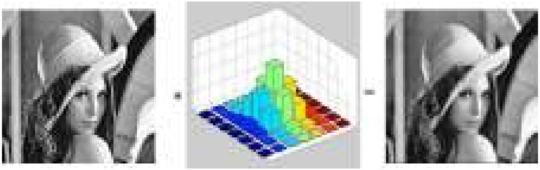


AVERAGE (5 × 5)

$$* \frac{1}{25} \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix} =$$

Filter Examples - 3

Gaussian - $\sim N(0, \sigma)$



Gaussian vs Average

Smoothing

Generally expect

- Pixels to “be like” neighbours
- Noise independent from pixel to pixel

Implies

- Smoothing suppress noises
- Appropriate noise model (?)

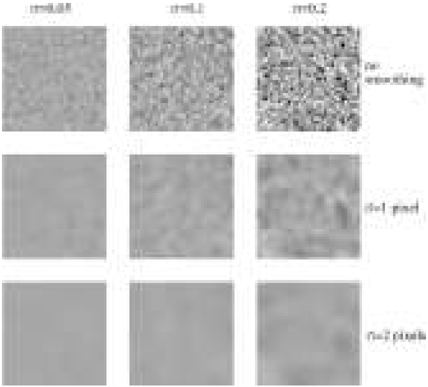


Image Gradient - 1

Horizontal derivatives (∇I_x)

*

1	0	-1
1	0	-1
1	0	-1

=

Image Gradient - 2

Vertical derivatives (∇I_y)

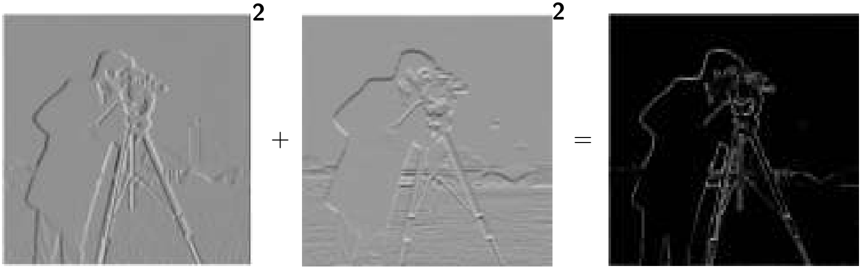
$*$

1	1	1
0	0	0
-1	-1	-1

$=$

Rough edge detector

$$\underline{\nabla I_x^2 + \nabla I_y^2}$$



then apply threshold...

Outline

Canny Edge Detector

Criterion

- *Good detection*: minimize probability of false positive and false negatives
- *Good localization*: Edges as closest as possible to the true edge
- *Single response*: Only one point for each edge point (thick $= 1$)

Procedure

- Smooth by Gaussian ($S = I * G(\sigma)$)
- Compute derivatives (∇S_x, ∇S_y)

  Alternative in one step: Filter with derivative of Gaussian
- Compute magnitude and orientation of gradient

  $(\|\nabla S_x\| = \sqrt{\nabla S_x^2 + \nabla S_y^2},\ \theta_{\nabla S} = \text{atan2}\,\nabla S_y, \nabla S_x)$
- Non maxima suppression

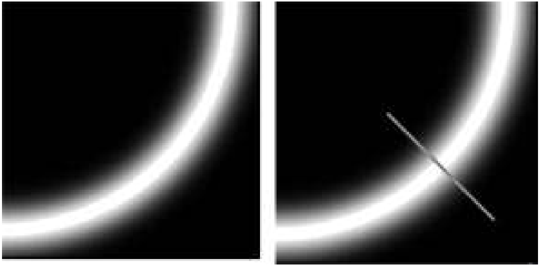
  Search for local maximum in the gradient direction $\theta_{\nabla S}$
- Hysteresis threshold

  Weak edges (between the two thresholds) are edges

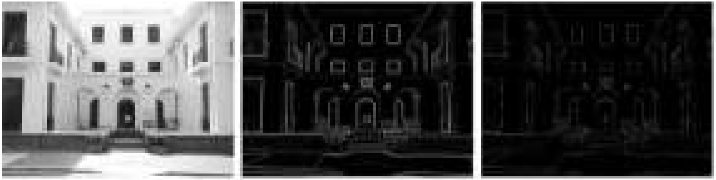
  if connected to strong edges (greater than high threshold)

Canny Edge Detector - Non Maxima Suppression

Non Maxima Suppression



Example



Original image   Gradient magnitude   Non-maxima suppressed

Canny Edge Detector - Hysteresis threshold

Hysteresis threshold example

Lines from edges

How to find lines?

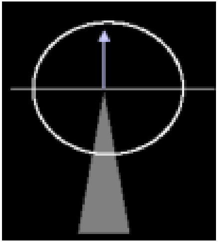
- *Hough Transformation* after Canny edges extraction
- Use a *voting procedure*
- Generally find imperfect instances of a shape class
- Classical Hough Transform for lines detection
- Later extended to other shapes (most common: circles, ellipses)

Example

Corners - Harris and Shi Tomasi

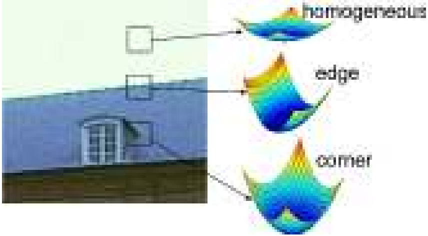
Edge intersection

- At intersection point gradient is ill defined
- Around intersection point gradient changes in "all" directions
- It is a "good feature to track"



Corner detector

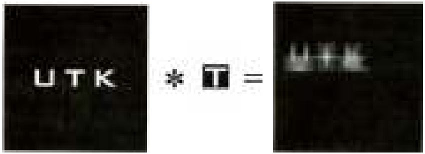
- Examine gradient over window
- $C = \sum_w \sum \begin{bmatrix} \nabla I_x^2 & \nabla I_x \nabla I_y \\ \nabla I_x \nabla I_y & \nabla I_y^2 \end{bmatrix}$
- Shi-Tomasi: corner if $\min \text{eigenvalue}(C) > T$
- Harris: approximation of eigenvalues



Template matching - Patch

Filtering with a template

- Correlation between template (patch) and image
- Maximum where template matches
- Alternatives with normalizations for illumination compensation, etc.



Good features

- On corners: higher repeatability (homogeneous zone and edges are not distinctive)

Template matching - SIFT

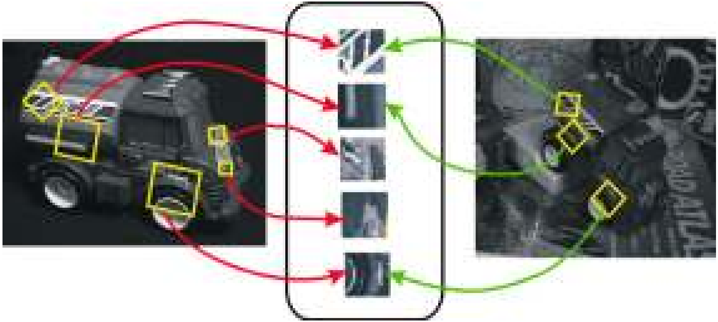
Template matching issues

- Rotations
- Scale change

SIFT

- Scale Invariant Feature Transform
- Alternatives descriptor to patch
- Performs orientation and scale normalization
- *See also SURF (Speeded Up Robust Feature)*

SIFT example



D. Lowe - “Object recognition from local scale-invariant features” - 1999

Outline

Camera matrix - 1

Given

- $\mathbf{P} = \begin{bmatrix} 122.5671 & -320.0000 & -102.8460 & 587.3835 \\ -113.7667 & 0.0000 & -322.2687 & 350.6050 \\ 0.7660 & 0 & -0.6428 & 4.6711 \end{bmatrix}$
- $f_x = f_y = 320$
- $c_x = 160$
- $c_y = 120$

Questions

- Where is the camera in the world?
- Compute the coordinate of the vanishing point of x, y plane in the image
- Where is the origin of the world in the image?
- Write the parametric 3D line of the principal axis in world coordinates
- ...

Camera matrix - 2

Where is the camera in the world?

- $\mathbf{P} = \begin{bmatrix} \mathbf{KR} & \mathbf{Kt} \end{bmatrix}$
- $\mathbf{K} = \begin{bmatrix} 320 & 0 & 1600 & 320 & 1200 & 0 & 1 \end{bmatrix}$
- $\mathbf{R} = \mathbf{K}^{-1}\mathbf{P}(1:3, 1:3)$
- $\mathbf{t} = \mathbf{K}^{-1}\mathbf{P}(1:3, 4)$
- $\mathbf{T}_{WC}^{(W)} = \begin{bmatrix} \mathbf{R} & \mathbf{t} \\ \mathbf{0} & 1 \end{bmatrix}^{-1}$

  P contains the world wrt camera
- Camera is at $[-4, -0.5, 2.5]$
- Rotation around axis x, y, z is $[-130°, 0.0°, -90°]$
- To be more clear, remove the rotation of camera reference frame
- $\mathbf{T}_{WC}^{(W)}\,\mathbf{R}_z(90°), \mathbf{R}_y(-90°)$

  rotation around axis x, y, z is $[0°, 40.0°, 0°]$

Camera matrix - 3

Vanishing point of x, y plane in the image

- $\mathbf{v}_x = \mathbf{P}\left[1, 0, 0, 0\right]^T \equiv \left[160.0,\ -148.5, 1\right]^T$
- $\mathbf{v}_y = \mathbf{P}\left[0, 1, 0, 0\right]^T \equiv \left[1,\ 0, 0\right]^T$ (improper point)
- Remember: they are the 1^{st} and 2^{nd} column of $\mathbf{P}$

Where is the origin of the world in the image?

- $\mathbf{o} = \mathbf{P}\left[0, 0, 0, 1\right]^T \equiv \left[125.75,\ 75.05, 1\right]^T$
- Remember: it is the 4^{st} column of $\mathbf{P}$

Principal axis in world coordinates

- $\mathbf{P} = \left[\mathbf{M} \quad \mathbf{m}\right]$
- $\mathbf{O}^{(W)} = -\mathbf{M}^{-1}\mathbf{m} = \left[-4, -0.5, 2.5\right]^T$
- $\mathbf{d}^{(W)} = \mathbf{M}^{-1}\left[c_x, c_y, 1\right]^T = \left[0.766, 0, -0.6428\right]^T$
- $\mathbf{a}^{(W)} = \mathbf{O}^{(W)} + \lambda\mathbf{d}^{(W)}$

Camera matrix - 5

Questions

- Where is the intersection between principal axis and the floor?
- Calculate the field of view of the camera (image size is 320×240)

  i.e., the portion of the plane imaged by the camera

Camera matrix - 6

Intersection between principal axis and the floor

- $\mathbf{a}^{(W)} = \mathbf{O}^{(W)} + \lambda \mathbf{d}^{(W)}$
- $\mathbf{a}_{z=0}^{(W)} = \left[X, Y, 0\right]^T$
- $\lambda_{z=0} = -\mathbf{O}_z^{(W)} / \mathbf{d}_z^{(W)} = 3.89$
- $\mathbf{a}_{z=0}^{(W)} = \left[-1.02, -0.5, 0\right]^T$

Field of view (1)

- $\mathbf{a}_1^{(W)} = \mathbf{O}^{(W)} + \lambda_1 \mathbf{M}^{-1} \left[0, 0, 1\right]^T$
- $\mathbf{a}_2^{(W)} = \mathbf{O}^{(W)} + \lambda_2 \mathbf{M}^{-1} \left[320, 0, 1\right]^T$
- $\mathbf{a}_3^{(W)} = \mathbf{O}^{(W)} + \lambda_3 \mathbf{M}^{-1} \left[0, 240, 1\right]^T$
- $\mathbf{a}_4^{(W)} = \mathbf{O}^{(W)} + \lambda_4 \mathbf{M}^{-1} \left[320, 240, 1\right]^T$
- calculate λ_i such that $\mathbf{a}_i^{(W)}$ has $z = 0$
- ...

Camera matrix - 7

Field of view (2)

- Transformation between plane $z = 0$ and image plane is a 2D homography
- Consider $\mathbf{p}_{z=0} = \left[x, y, 0, 1\right]^T$;
- Projection $\mathbf{P}\mathbf{p}_{z=0}$
- Notice that $\mathbf{p}_{z=0} = \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{\mathbf{W}} \left[x, y, 1\right]^T$
- $\mathbf{H} = \mathbf{PW}$ is the homography

  maps points (x, y) on the $z = 0$ plane to the image
- $\mathbf{H}' = \mathbf{H}^{-1}$ is the inverse homography

  maps image points (u, v) to the $z = 0$ plane

Camera matrix - 8

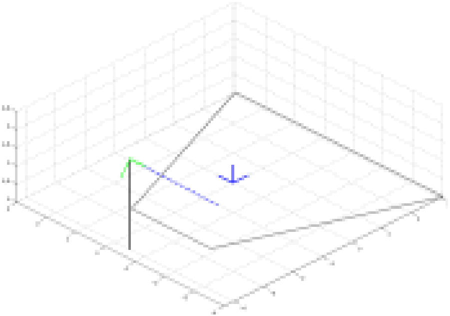
FIELD OF VIEW (2) (CONTINUE)

- $\mathbf{H} = \begin{bmatrix} 122.56 & -320.00 & 587.38 \\ -113.76 & 0.00 & 350.60 \\ 0.76 & 0 & 4.67 \end{bmatrix}$
- $\mathbf{p}_{1_{z=0}} = \mathbf{H}' \left[0, 0, 1\right]^T$
- $\mathbf{p}_{2_{z=0}} = \mathbf{H}' \left[320, 0, 1\right]^T$
- $\mathbf{p}_{3_{z=0}} = \mathbf{H}' \left[0, 240, 1\right]^T$
- $\mathbf{p}_{4_{z=0}} = \mathbf{H}' \left[320, 240, 1\right]^T$

Camera matrix - 8

Field of view (2) (continue)

- $\mathbf{H} = \begin{bmatrix} 122.56 & -320.00 & 587.38 \\ -113.76 & 0.00 & 350.60 \\ 0.76 & 0 & 4.67 \end{bmatrix}$
- $\mathbf{p}_{1\,z=0} = \mathbf{H}' \left[0, 0, 1\right]^T$
- $\mathbf{p}_{2\,z=0} = \mathbf{H}' \left[320, 0, 1\right]^T$
- $\mathbf{p}_{3\,z=0} = \mathbf{H}' \left[0, 240, 1\right]^T$
- $\mathbf{p}_{4\,z=0} = \mathbf{H}' \left[320, 240, 1\right]^T$



The 3D world with the camera reference system (green), the world reference system (blue), the principal axis (dashed blue) and the Field of view (FoV) (grey)

Camera matrix - 9

Questions

- There is a "flat robot" moving on the floor, imaged by the camera
- Two distinct and coloured point are drawn on the robot.
  $$\mathbf{p}_1^{(R)} = \left[-.3, 0\right]^T, \ \mathbf{p}_2^{(R)} = \left[.3, 0\right]^T$$
- Could you calculate the robot position and orientation?

Camera matrix - 10

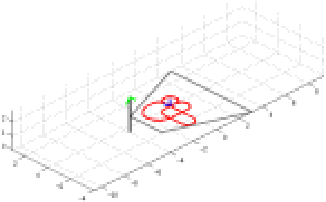
Robot pose

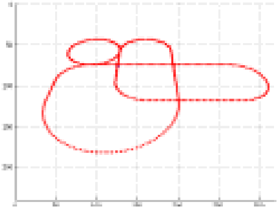
- Call $\mathbf{p}'_1$, $\mathbf{p}'_2$ the points in the image
- $\mathbf{p}_{1_{z=0}} = \mathbf{H}'\mathbf{p}'_1$
- $\mathbf{p}_{2_{z=0}} = \mathbf{H}'\mathbf{p}'_2$
- Position: $\frac{1}{2}(\mathbf{p}_{1_{z=0}} + \mathbf{p}_{2_{z=0}})$
- $\mathbf{d} = \mathbf{p}_{2_{z=0}} - \mathbf{p}_{1_{z=0}}$
- Orientation: $\text{atan2}(\mathbf{d}_y, \mathbf{d}_x)$

Camera matrix - 10

Robot pose

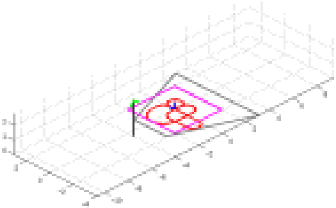
- Call $\mathbf{p}'_1$, $\mathbf{p}'_2$ the points in the image
- $\mathbf{p}_{1_{z=0}} = \mathbf{H}'\mathbf{p}'_1$
- $\mathbf{p}_{2_{z=0}} = \mathbf{H}'\mathbf{p}'_2$
- Position: $\frac{1}{2}(\mathbf{p}_{1_{z=0}} + \mathbf{p}_{2_{z=0}})$
- $\mathbf{d} = \mathbf{p}_{2_{z=0}} - \mathbf{p}_{1_{z=0}}$
- Orientation: $\text{atan2}(\mathbf{d}_y, \mathbf{d}_x)$





A complete execution with a robot trajectory depicted in the world (top) and in the image (bottom)

Camera matrix - 11



A complete execution with a robot trajectory depicted in the world (top) and in the image (bottom). A Square is drawn on the floor to check correctness of the calculated vanishing point x (see previous questions)